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The use of force sensors and a computer system to introduce the concept of inertia at a school

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Abstract
A classical experiment used to introduce the concept of body inertia, breaking of a thread below and above a hanging weight, is described mathematically and presented in a new way, using force sensors and a computer system.

Introduction

To introduce the concept of inertia (body inertia, matter inertia) [1] at school one can use a well-known, classical experiment, breaking of a thread below and above a hanging weight (figure 1). Both threads, i.e. those above and below the weight, are identical; however, which thread is broken when pulling downward on the lower thread depends on how the pulling process is conducted. Slow increase of the force acting on the lower thread results in breaking of the upper thread. This result is expected because, apart from the pulling force, an additional force (the weight) acts on the upper thread. However, abrupt increase of the force (a jerk) acting on the lower thread causes, unexpectedly, breaking of this thread. This observation may be surprising for students and can be a starting point for a discussion of the concept of inertia.

All bodies are elastic to some extent. Pulling on the upper thread results in its slight elongation. One has to move the weight slightly in order to elongate the thread, which requires time. Even a very large force acting on the lower thread will not cause an immediate shift of the weight. Before the shift is completed, the lower thread can break.

A mathematical description of the weight's motion allows one to understand the conditions for breaking the chosen thread.

Mathematical description of the breaking of the thread above or below the hanging weight

The observed phenomenon can be described assuming weightlessness and elasticity of the threads and a linear, proportional to time t, increase of the force F acting on the lower thread,

$$F = w \cdot t. \tag{1}$$

The coefficient w describes the rate of increase of the acting force.

The motion of the weight, and hence the elongation of the upper thread, x, can be described by the equation

$$m\ddot{x} = w \cdot t + mg - kx, \tag{2}$$

where m is the mass of the weight, k is the elasticity coefficient of the thread and g is the gravitational acceleration. The solution of the

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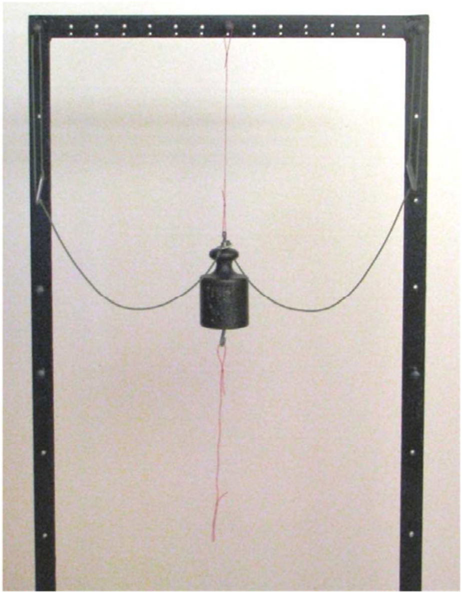

Figure 1. The experimental set-up. A weight is hanging on a thread, with a second thread attached below. The thin ropes attached to the weight are intended to catch the weight after breaking of the upper thread.

above equation takes the form [2]

$$x(t) = \frac{w}{k}\left(t + \frac{mg}{w} - \sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}t\right)\right). \quad (3)$$

Therefore, the force acting on the upper thread, F_g, can be expressed as

$$F_g = kx = wt + mg - w\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}t\right). \quad (4)$$

We are interested in how the force F_g depends on the force F acting on the lower thread (increasing linearly with t). Using (1) to eliminate t, one obtains

$$F_g = F + mg - w\sqrt{\frac{m}{k}}\sin\left(\sqrt{\frac{k}{m}}\frac{F}{w}\right). \quad (5)$$

F_g is smaller than the sum of mg and the force acting on the lower thread, F, by a component proportional to the coefficient w. This is just a consequence of the limited time that is necessary to shift the weight and to stretch the upper thread.

In the case of a slow increase of the force acting on the lower thread ($w \to 0$), the contribution containing w can be neglected, so the force acting on the upper thread is the sum of mg and the force acting on the lower thread,

$$F_g = F + mg. \quad (6)$$

F_g is larger than the force acting on the lower thread and, as observed in experiment, the upper thread gets broken. In the case of an abrupt jerk ($w \to \infty$), F_g can be smaller than F (see (5)), which gives the chance to break the lower thread.

One can also determine a 'threshold' rate of increase w_g of the force F; that is, a rate of increase such that both threads break at the same time (this indeed sometimes happens accidentally in experiments).

This allows a better understanding of the conditions under which each thread is broken. Approximation of the sine function in (5) with two components of a Taylor series gives

$$F_g = mg + \frac{1}{6}\frac{k}{m}\frac{F^3}{w^2}. \quad (7)$$

At the moment of simultaneous breaking of both threads, the force F applied to the lower thread and the force F_g acting on the upper thread are equal to F_z, the force necessary to break a thread,

$$F_g = F = F_z. \quad (8)$$

By substituting this condition into (7), one obtains

$$F_z = mg + \frac{1}{6}\frac{k}{m}\frac{F_z^3}{w_g^2}, \quad (9)$$

which makes it possible to calculate the 'threshold' rate of increase w_g,

$$w_g^2 = \frac{1}{6}\frac{k}{m}\frac{F_z^3}{(F_z - mg)}. \quad (10)$$

For $w < w_g$ the upper thread is broken and for $w > w_g$ the lower thread is broken.

The value of w_g depends on the force F_z required to break a thread (figure 2). One can prove, by analysing the extremum of the function (10), that the easiest way to break the lower thread (the lowest rate of increase that is sufficient to break a lower thread) is for such thread for which $F_z = \frac{3}{2}mg$. This corresponds to $F_z = 15$ N on the plot in figure 2.

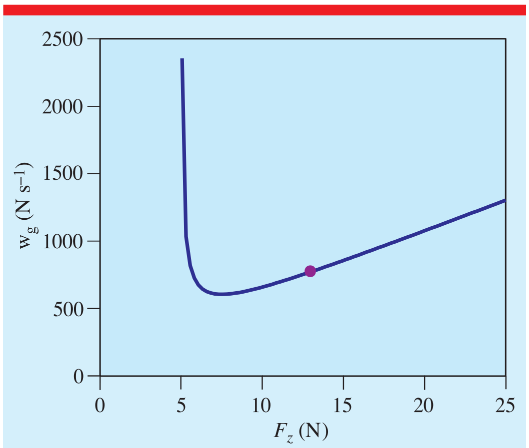

Figure 2. The dependence of the 'threshold' rate of increase of the force applied to lower thread w_g on the force necessary to break the thread F_z. Calculations were carried out assuming a 1 kg weight, as used in the experiment. The black point corresponds to the force necessary to break the actual thread used in the experiment.

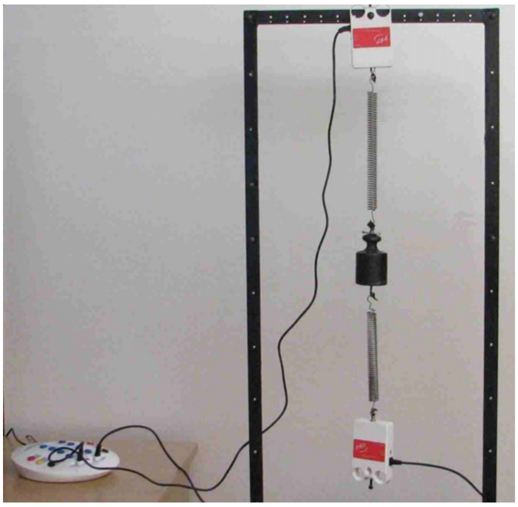

Figure 4. Experimental set-up with springs used instead of threads.

Application of a computer system

The classical experiment allows one to obtain only indirect information on the forces acting on the threads. When we observe breaking of a thread we conclude that the force F_z necessary to break the thread was surpassed. By applying force sensors and the computer system COACH [3, 4] to aid the physics experiments it is possible to observe directly the time dependence of the acting forces (figure 3). In the case of a slow increase of the force F acting on the lower thread, the force F_g (larger by approximately mg) (figure 3(a)) acts on the upper thread. F_g reaches the value necessary to break a thread first, so the upper thread breaks. For an abrupt jerk, the value of the force F applied to the lower thread increases faster than the force F_g acting on the upper thread (figure 3(b)). The lower thread breaks, and the weight undergoes damped harmonic oscillations on the upper thread. By measuring the period of

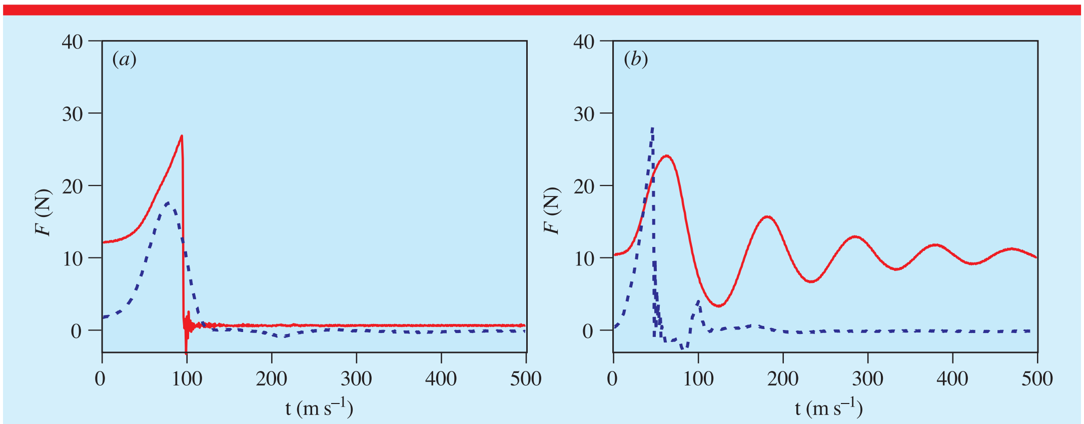

Figure 3. The time dependences of the forces acting on the lower thread (broken line) and the upper thread (solid line) for two different rates of increase of the applied force.

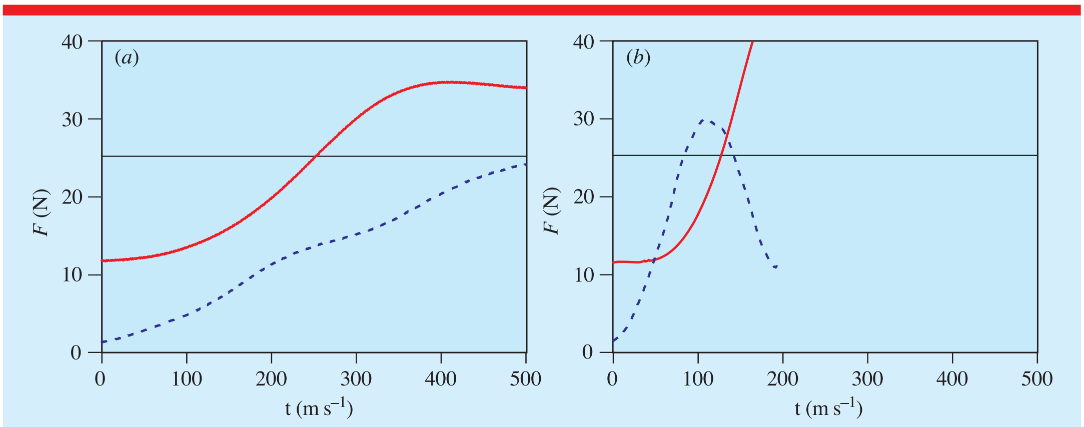

Figure 5. The time dependences of the forces acting on the lower spring (broken line) and the upper spring (solid line) for two different rates of increase of the applied force. Additionally, the value of the force necessary to break a hypothetical thread is marked.

these oscillations, knowing the mass, it is possible to determine the coefficient of elasticity of the thread, k, using the relation

$$T = 2\pi\sqrt{\frac{m}{k}}. \qquad (11)$$

Additionally, by applying a computer system it is possible to replace the threads (which are difficult to replace quickly) with springs (figure 4), and the value of the force necessary to break a fictitious thread can be marked on the plot (figure 5). Such an experimental set-up allows students to conduct many observations in a short time for different rates of increase of the force.

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